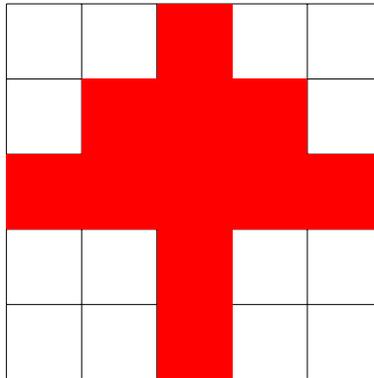


**Time limit:** 15 minutes.

**Instructions:** This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

**No calculators.**

- Find the smallest positive integer  $n$  such that there exists a prime  $p$  where  $p$  and  $p + 10$  both divide  $n$  and the sum of the digits of  $n$  is  $p$ .
- Every cell in a  $5 \times 5$  grid of paper is to be painted either red or white with equal probability. An edge of the paper is said to have a “tree” if the set of cells depicted in the diagram below are all painted red when the paper is rotated so that the edge lies at the bottom. Given that at least one edge of the paper has a tree, what is the expected number of edges that have a tree?



- What is the least positive integer  $x$  for which the expression  $x^2 + 3x + 9$  has 3 distinct prime divisors?