Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise.
No calculators.

1. There exists a unique real value of x such that
$$(x+\sqrt{x})^2 = 16.$$
Compute x.

2. Compute the number of values of x in the interval $[-11\pi, -2\pi]$ that satisfy $\frac{5\cos(x)+4}{5\sin(x)+3} = 0$.

3. Nathan has discovered a new way to construct chocolate bars, but it's expensive! He starts with a single 1×1 square of chocolate and then adds more rows and columns from there. If his current bar has dimensions $w \times h$ (w columns and h rows), then it costs w^2 dollars to add another row and h^2 dollars to add another column. What is the minimum cost to get his chocolate bar to size 20×20?

4. If the sum of the real roots x to each of the equations
$$2^{2x} - 2^{x+1} + 1 - \frac{1}{k^2} = 0$$
for $k = 2, 3, \ldots, 2023$ is N, what is 2^N?

5. Suppose $\alpha, \beta, \gamma \in \{-2, 3\}$ are chosen such that
$$M = \max_{x\in\mathbb{R}} \min_{y\in\mathbb{R}_{\geq 0}} \alpha x + \beta y + \gamma xy$$
is finite and positive (note: $\mathbb{R}_{\geq 0}$ is the set of nonnegative real numbers). What is the sum of the possible values of M?

6. What is the area of the figure in the complex plane enclosed by the origin and the set of all points $\frac{1}{z}$ such that $(1-2i)z + (-2i-1)\overline{z} = 6i$?

7. Consider a sequence $F_0 = 2$, $F_1 = 3$ that has the property $F_{n+1}F_{n-1} - F_n^2 = (-1)^n \cdot 2$. If each term of the sequence can be written in the form $a \cdot r_1^n + b \cdot r_2^n$, what is the positive difference between r_1 and r_2?

8. If x and y are real numbers, compute the minimum possible value of
$$\frac{4xy(3x^2+10xy+6y^2)}{x^4+4y^4}.$$

9. Let x, y, z be nonzero numbers, not necessarily real, such that
$$(x-y)^2 + (y-z)^2 + (z-x)^2 = 24yz$$
and
$$\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = 3.$$
Compute $\frac{x^2}{yz}$.

10. Suppose that $p(x), q(x)$ are monic polynomials with nonnegative integer coefficients such that

$$\frac{1}{5x} \geq \frac{1}{q(x)} - \frac{1}{p(x)} \geq \frac{1}{3x^2}$$

for all integers $x \geq 2$. Compute the minimum possible value of $p(1) \cdot q(1)$.