1. How many possible values of n are there from 1 to 50 such that $xy - 4x - 6y = n$ has 8 ordered pair integer solutions (x, y)?

   Answer: 16

   Solution: We can rewrite the equation as $(x-6)(y-4) = 24 + n$. We see that $24 + n$ will be between 25 and 74, inclusive. Each solution (x, y) corresponds to a way of factoring $24 + n$. The order of the factors matter and the factors can be negative. From this, we conclude that $24 + n$ must have $8/2 = 4$ factors. So, $24 + n = p^3$ or $24 + n = pq$ where p, q are prime (p and q are distinct). For the first case, we only have the possibility $3^3 = 27$. For the second case, we want to count how many numbers are of the form pq between 25 and 74. Without loss of generality, let $p < q$. If $p = 2$, q can be $13, 17, 19, 23, 29, 31$, or 37. If $p = 3$, q can be $11, 13, 17, 19$, or 23. If $p = 5$, q can be $7, 11$, or 13. If $p > 5$, there is no q such that $pq \le 74$. Adding up the total number of possibilities for the first case and second case gives $1 + 7 + 5 + 3 = \boxed{16}$.

2. Gordon and Joey each have two fair 8-sided dice where each side is labeled with a positive integer. Gordon's two dice are each labeled with the numbers from 1 to 8 exactly once. One of Joey's dice has a side labeled with the number 10 while the other 15 sides have labels in the positive integers. Gordon and Joey each roll both of their dice and take the sum of two numbers on the top face of each dice. For any number n, the probability that the sum of Gordon's numbers equaling n is the same as the probability that the sum of Joey's numbers equaling n. What is the product of the numbers on Joey's dice that does not have a 10 on one of its sides?

   Answer: 8640

   Solution: We can represent the possible sums of Gordon's dice rolls with the generating function

   $$(x^1 + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8)^2 = x^2(x+1)^2(x^2+1)^2(x^4+1)^2.$$

   Since each side of any dice can only contain positive numbers, Joey's die that contains a 10 must be represented by the generating function $x(x+1)(x^4+1)^2$ and have sides labeled with $\{1, 2, 5, 5, 6, 6, 9, 10\}$. This means that Joey's other die must be represented by the generating function $x(x+1)(x^2+1)^2$. Therefore, this die has sides labeled with $\{1, 2, 3, 3, 4, 4, 5, 6\}$, which yields a product of $\boxed{8640}$.

3. Let k_1, k_2, k_3, k_4 be arbitrary circles, such that k_1 is externally tangent to k_2 and k_4 at points A and D, and k_3 is externally tangent to k_2 and k_4 at points B and C. $\angle BCD = 60°$. Find the measure of $\angle BAD$ (in degrees).

   Answer: 120°

   Solution: Let I be an inversion with center at A and arbitrary radius r. Therefore, we have that under I, k_1 and k_2 map to parallel lines $l_1 \| l_2$, since k_1 and k_2 intersect only at point A. k_3 and k_4 do not contain A, therefore they map to circles k_3' and k_4'. We have that k_1 and k_4 have one common point (D, therefore l_1 is tangent to k_4' at D', and analogously l_2 is tangent to k_3' at B'. Since k_3 and k_4 are tangent at C, k_3' and k_4' are tangent at C', and because $l_1 \| l_2$, we have that B', C', D' are collinear. Therefore, reversing the inversion, $ABCD$ is cyclic, and therefore $\angle BAD = 180 - \angle BCD = 120°$, as desired.