Time limit: 15 minutes.
Instructions: This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.
No calculators.

1. How many 4 element subsets of $\{0, 1, 2, \dots, 20\}$ contain their sum modulo 21?

2. Let a, b, c be the solutions to $x^3 + 3x^2 - 1 = 0$. Define $S_n = a^n + b^n + c^n$. Given that there are integers $0 \leq i, j, k \leq 36$ such that $S_n \equiv i^n + j^n + k^n \pmod{37}$ for all positive integer n, determine the product ijk.

3. Five lilypads lie in a line on a pond. At first, a frog sits on the third lilypad. Then, each minute there is a $\frac{1}{2}$ probability that the frog jumps to the lilypad to its left and $\frac{1}{2}$ probability that it jumps to its right. If the frog jumps to the left from the leftmost lilypad or right from the rightmost lilypad, it will fall in the pond and stay there forever. Compute the probability that the frog is not in the pond after 14 minutes have passed.