Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise.
No calculators.

1. Compute
$$\frac{5+\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\frac{7+\sqrt{12}}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{63+\sqrt{992}}{\sqrt{31}+\sqrt{32}}.$$

2. Find the sum of the solution(s) x to the equation
$$x=\sqrt{2022+\sqrt{2022+x}}. \quad (1)$$

3. Compute $\left\lfloor \frac{1}{\frac{1}{2022}+\frac{1}{2023}+\cdots+\frac{1}{2064}} \right\rfloor$.

4. Let the roots of
$$x^{2022}-7x^{2021}+8x^2+4x+2$$
be $r_1, r_2, \cdots, r_{2022}$, the roots of
$$x^{2022}-8x^{2021}+27x^2+9x+3$$
be $s_1, s_2, \cdots, s_{2022}$, and the roots of
$$x^{2022}-9x^{2021}+64x^2+16x+4$$
be $t_1, t_2, \cdots, t_{2022}$. Compute the value of
$$\sum_{1\le i,j\le 2022} r_i s_j + \sum_{1\le i,j\le 2022} s_i t_j + \sum_{1\le i,j\le 2022} t_i r_j.$$

5. x, y, and z are real numbers such that $xyz = 10$. What is the maximum possible value of $x^3y^3z^3 - 3x^4 - 12y^2 - 12z^4$?

6. Compute
$$\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right).$$

7. Let $M = \{0, 1, 2, ..., 2022\}$ and let $f : M \times M \to M$ such that for any $a, b \in M$,
$$f(a, f(b, a)) = b$$
and $f(x, x) \neq x$ for each $x \in M$. How many possible functions f are there (mod 1000)?

8. For all positive integers $m > 10^{2022}$, determine the maximum number of real solutions $x > 0$ of the equation $mx = \lfloor x^{11/10} \rfloor$.

9. Let $P(x) = 8x^3 + ax^2 + bx + 1$ for $a, b \in \mathbb{Z}$. It is known that P has a root $x_0 = p + \sqrt{q} + \sqrt[3]{r}$, where $p, q, r \in \mathbb{Q}, q \geq 0$; however, P has no *rational* roots. Find the smallest possible value of $a + b$.

10. Let $f^1(x) = x^3 - 3x$. Let $f^n(x) = f(f^{n-1}(x))$. Let $\mathcal{R}$ be the set of roots of $\frac{f^{2022}(x)}{x}$. If

$$\sum_{r \in \mathcal{R}} \frac{1}{r^2} = \frac{a^b - c}{d}$$

for positive integers a, b, c, d, where b is as large as possible and c and d are relatively prime, find $a + b + c + d$.