1. Let a be the base 10 integer equivalent to 2021_{20} and let b the base 10 integer equivalent to 2021_{21}. Compute $b - a$.

   Answer: 2524

   Solution: Converting to base 10,

$$\begin{aligned} b - a &= (2 \cdot 21^3 + 2 \cdot 21 + 1) - (2 \cdot 20^3 + 2 \cdot 20 + 1) \\ &= 2(21^3 - 20^3) + 2(21 - 20) \\ &= 2(1261) + 2(1) \\ &= \boxed{2524}. \end{aligned}$$

2. Let f be a function such that for any positive integer n, $f(n)$ is equal to the median of the positive factors of n. Compute the sum of all positive integers n such that $20 < f(n) < 21$.

   Answer: 2856

   Solution: Bash pairs that sum to 41 to get the solution set $\{148, 310, 348, 390, 408, 414, 418, 420\}$ which sums to 2856.

3. Let k be a randomly chosen positive divisor of 20!. What is the probability that k can be written as $a^2 + b^2$ for some integers a and b?

   Answer: $\mathbf{\frac{5}{54}}$

   Solution: The choice k can be written as the sum of two squares if and only if every prime $p \equiv 3 (\text{mod } 4)$ appears to an even power in the prime factorization of k. The primes $p \leq 20$ with $p \equiv 3 (\text{mod } 4)$ are 3, 7, 11, and 19. Also, $20! = 3^8 \cdot 7^2 \cdot 11^1 \cdot 19^1 \cdot k$ where k is not divisible by any of these primes. Therefore, a randomly positive divisor of 20! has prime factorization $3^a \cdot 7^b \cdot 11^c \cdot 19^d \cdot l$ where l is not divisible by any of these primes, $a \in [0, 8]$, $a \in [0, 2]$, $a \in [0, 1]$, $a \in [0, 1]$. Furthermore a, b, c, and d take on the possible values with equally likelihood and independently of each other. It suffices to compute the probability that a, b, c, and d are all even which is $\left(\frac{5}{9}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{5}{54}}$.