1. The values E, R, I, C (not necessarily distinct) are chosen such that $E \cdot R \cdot I \cdot C = 1600$. How many quadruples (E, R, I, C) exist?

   Answer: 840

   Solution: Consider how we can distribute powers of 2, 5 to each of the four values; these powers can be independently assigned. Hence, the problem is equivalent to computing the number of ways to distribute 2^6 to each of the four values, and then computing the number of ways to distribute 5^2 to each of the four values.

   To distribute the powers of 2, we observe that this is equivalent to distributing 6 identical balls to 4 indistinguishable urns, which is equivalent to placing 3 dividers between 6 items. There are hence $\binom{9}{3} = 84$ ways of distributing the powers of 2. Similarly, there are $\binom{5}{3} = 10$ ways of distributing the powers of 5. Hence, there are a total of $84 \cdot 10 = 840$ ways of distributing the factors between E, R, I, C.

2. Three spheres are centered at the vertices of a triangle in the horizontal plane and are tangent to each other. The triangle formed by the uppermost points of the spheres has side lengths 10, 26, and $2\sqrt{145}$. What is the area of the triangle whose vertices are at the centers of the spheres?

   Answer: $5\sqrt{119}$

   Solution: The line segment connecting the uppermost points of two spheres with radii r_1 and r_2, where $r_1 > r_2$, is the hypotenuse of a right triangle with legs of length $r_1 - r2$ and $r_1 + r_2$. So, the length of such a line segment is $\sqrt{(r_1 - r_2)^2 + (r_1 + r_2)^2} = \sqrt{2r_1^2 + 2r_2^2}$. We can set up a system of equations to solve for the radii of the spheres.

$$2r_1^2 + 2r_2^2 = 100$$

$$2r_2^2 + 2r_3^2 = 676$$

$$2r_2^3 + 2r_1^2 = 580$$

   The solution to this system is $r_1 = 1$, $r_2 = 7$, and $r_3 = 17$. The triangle whose vertices are the centers of the spheres simply has side lengths of $r_1 + r_2$, $r_2 + r_3$, and $r_3 + r_1$, which are equal to 8, 24, and 18. We can find the area of this triangle using Heron's Formula: $\sqrt{\frac{8+18+24}{2} \cdot \frac{-8+18+24}{2} \cdot \frac{8-18+24}{2} \cdot \frac{8+18-24}{2}} = \boxed{5\sqrt{119}}$.

3. Let A be the number of positive integers less than 2019 where x^{2020} has last digit 1. Let B be the number of positive integers less than 2019 where x^{2019} has last digit 6. What is $A - B$?

   Answer: 602

   Solution: We will compute A and B with Fermat's little theorem. Note that $\varphi(10) = 4$. So, if $\gcd(x, 10) = 1$ then $x^{2020} \equiv 1 \mod 10$. So, this is equivalent to finding the last digits 1,3,7,9. So, there are $A = 803$.

   Now if $\gcd(x, 10) \neq 1$, then we have $x^{2017} \equiv x \mod 10$. Hence, we need to figure out when $x^3 \equiv 6 \mod 10$ which occurs when $x \equiv 6 \mod 10$. So, $B = 201$. Hence $A - B = \boxed{602}$.