Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. If $f(x) = nx$, $g(x) = e^{2x}$, and $h(x) = g(f(x))$, find n such that $h'(0) = 100$.

2. Farmer Joe will plant carrots to cover a rectangle in the first quadrant with a vertex at the origin and sides parallel to the x and y axes. However, he can not grow carrots on his neighbor's land. If the border between his and his neighbor's land is along the curve $y = -\ln(2x)$, what is the maximum area of carrotland Farmer Joe can create?

3. For all θ from 0 to 2π, Annie draws a line segment of length θ from the origin in the direction of θ radians. What is the area of the spiral swept out by the union of these line segments?

4. The Chebyshev Polynomials are defined as
$$T_n(x) = \cos(n\cos^{-1}(x)),$$
for $n = 0, 1, 2, ...$ Compute the following infinite series:
$$\sum_{n=1}^{\infty} \int_{-1}^{1} T_{2n+1}(x)dx.$$
If the series diverges, your answer should be "D."

5. What is
$$(2020)^2 + \frac{(2021)^2}{1!} + \frac{(2022)^2}{2!} + \frac{(2023)^2}{3!} + \frac{(2024)^2}{4!} + \dots$$

6. Let us define the sequence $a_n = (-1)^n/(n)$. Now, we define the partial sums
$$A_N = \sum_{n=1}^{N} a_n.$$
What is the difference
$$\sum_{N=1}^{\infty} \left(A_N - \lim_{M\to\infty} A_M)\right)?$$

7. Define $f_1(x) = x$ and for every integer $n \geq 2$, we define $f_n(x) = x^{f_{n-1}(x)}$. Compute
$$\lim_{n\to\infty} \int_{e}^{2020} \frac{f_n'(x)}{f_n(x)f_{n-1}(x)\ln x} - \frac{f_{n-1}'(x)}{f_{n-1}(x)}dx.$$

8. Compute
$$\int_0^{\infty} \frac{dx}{x^4 - 6x^2 + 25}.$$

9. Define $a_n = \underbrace{\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}}_{\text{n square roots}}$. For example $a_1 = \sqrt{2}$ and $a_2 = \sqrt{2+\sqrt{2}}$. Find the value of

$$\lim_{n\to\infty} 4^n(2-a_n).$$

10. Let

$$I_m = \int_0^{2\pi} \sin(x)\sin(2x)\cdots\sin(mx)dx.$$

Find the sum of all integers $1 \le m \le 100$ such that $I_m \neq 0$.