Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Evaluate $\sqrt{2223^2 - 8888}$.

2. If a is the only real number that satisfies $\log_{2020} a = 202020 - a$ and b is the only real number that satisfies $2020^b = 202020 - b$, what is the value of $a + b$?

3. Two cars driving from city A to city B leave at the same time. The first car drives at some constant speed during the whole trip. The second car travels at a speed 12 km/hr slower than the first car until the halfway point between city A and city B. After the halfway point, the second car travels at a constant speed of 72 km/hr. Both cars end up reaching city B at the same time. Calculate the speed of the first car in km/hr, given that it was faster than 40 km/hr.

4. Find the value of bc such that $x^2 - x + 1$ divides $20x^{11} + bx^{10} + cx^9 + 4$.

5. Suppose $f(x)$ is a monic quadratic polynomial such that there exists an increasing arithmetic sequence $x_1 < x_2 < x_3 < x_4$ where $|f(x_1)| = |f(x_2)| = |f(x_3)| = |f(x_4)| = 2020$. Compute the absolute difference of the two roots of $f(x)$.

6. Let $f : A \to B$ be a function from $A = \{0, 1, \dots, 8\}$ to $B = \{0, 1, \dots, 11\}$ such that the following properties hold:
$$f(x + y \mod 9) \equiv f(x) + f(y) \mod 12$$
$$f(xy \mod 9) \equiv f(x)f(y) \mod 12$$
for all $x, y \in A$. Compute the number of functions f that satisfy these conditions.

7. Let a_n be a sequence where $a_0 = \sqrt{3}, a_1 = \sqrt{2}, a_3 = -1$ (not a_2) and $a_n = a_{n-1}a_{n-2} - a_{n-3}$ for $n \geq 3$. Compute a_{2020}.

8. For how many integers n with $3 \leq n \leq 2020$ does the inequality
$$\sum_{k=0}^{\lfloor (n-1)/4 \rfloor} \binom{n}{4k+1} 9^k > 3 \sum_{k=0}^{\lfloor (n-3)/4 \rfloor} \binom{n}{4k+3} 9^k$$
hold?

9. A sequence of numbers is defined by $a_0 = 2$ and for $i > 0$, a_i is the smallest positive integer such that $\sum_{j=0}^{i} \frac{1}{a_j} < 1$. Find the smallest integer N such that $\sum_{i=N}^{\infty} \frac{1}{\log_2(a_i)} < \frac{1}{2^{2020}}$.

10. Let $f(a, b)$ be a third degree two-variable polynomial with integer coefficients such that $f(a, a) = 0$ for all integers a and the sum
$$\sum_{\substack{a,b \in \mathbf{Z}^+ \\ a \neq b}} \frac{1}{2^{f(a,b)}}$$
converges. Let $g(a, b)$ be the polynomial such that $f(a, b) = (a - b)g(a, b)$. If $g(1, 1) = 5$ and $g(2, 2) = 7$, find the maximum value of $g(20, 20)$.