Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. In a school there are 47 tenth graders and 36 twelfth graders. Of these students 25 of them are born in the winter and 26 of the twelfth graders are not born in the winter. How many tenth graders were not born in winter?

2. Let $x = 1 - 3 + 5 - 7 + ... - 99 + 101$, and let $y = 2 - 4 + 6 - 8 + ... - 100$. Compute $y - x$.

3. In your drawer you have 23 green socks, 12 red socks, 42 blue socks, and 39 yellow socks. It is too dark to tell them apart. How many socks must you pull out to guarantee that you will have a green pair and a red pair?

4. The blue train heads north at 60 miles per hour from Blueville. The red train starts an hour later and heads south at 80 miles per hour from Redtown. If Redtown is 200 miles north of Blueville, what is the ratio of the distance the blue train travels to the distance the red train travels before the collision?

5. Maddy wants to create a 10 letter word with using only letters in her name. If she uses m M's, a A's, d D's, and y Y's where $m > a > d > y > 0$, what is $m \cdot a \cdot d \cdot d \cdot y$?

6. Harry has a chocolate kiss (cone-shaped chocolate) with radius 2 inches and height 4 inches. If he bites off a cone at the top of the kiss of height 2 inches, what is the volume of the remaining kiss?

7. Bob and Joe are running around a 500m track. Bob runs clockwise at 5 m/s and Joe runs counterclockwise at 10 m/s. They start at the same spot on the track and run for 10 minutes. How many times do they pass each other after they start running?

8. 4 people are sitting in a line. However, 2 people are best friends and must sit next to each other. How many possible ways can they sit?

9. What is the remainder when 2019^{2019} is divided by 7?

10. A rectangular soccer field has a diagonal of 29 and an area of 420. What is the perimeter of the field?

11. Let n be an integer such that $n^4 - 2n^3 - n^2 + 2n + 2$ is a prime number. What is the sum of all possible n?

12. A thief steals a watch and taunts the cop "an old man like you could never catch a kid like me!" 12 years later when the thief is caught, the thief and the cops ages sum to 72. At the time of the theft, the product of the thief and cops ages was a power of two. How old was the cop when he caught the thief?

13. The number N_b is the number such that when written in base b, it is 123. What is the smallest b such that N_b is a cube of a positive integer?

14. A cat chases a mouse down on the xy plane. The cat starts at the origin and the mouse at $(1, 0)$. The mouse runs straight towards the mouse hole at $(1, 3)$. The cat runs towards the place at which it will catch the mouse. If the cat runs at 5 units/sec and the mouse at 3 units/sec, how far away from the hole was the mouse when it was caught?

15. Find the number of two-digit positive integers that are divisible by the sum of their own digits.

16. A hexagon of side length $\sqrt{24}$ and a circle share the same center. The total area of the regions that are inside the circle and outside the hexagon is equal to the total area of the regions that are outside the circle and inside the hexagon. What is the square of the radius of the circle?

17. Let $ABCD$ be a square with points X and Y on BC and CD respectively. If $XY = 29$, $CY = 21$ and $BX = 15$, what is $\angle XAY$ in degrees?

18. Peter has 18 colored gumballs composed of 3 red, 4 blue, 5 yellow, and 6 green where same colored gumballs are indistinguishable. What is the probability that if he chooses four gumballs at random, the gumballs that he chooses consist of at least two colors and at most three colors?

19. How many rational numbers can be written in the form $\frac{a}{b}$ such that a and b are relatively prime positive integers and the product of a and b is $(25!)$?

20. In Smashville, there is one main straight highway running east-west between a gas station and a lake. Gina is driving along a scenic path that crosses the highway three times and can be described by a cubic polynomial. The crossings are respectively 1, 3, and 6 miles east down the highway from gas station. The distance between the scenic path and the highway is 18 miles when the path is directly south of the gas station. How far away from the gas station is Gina when she is 8 miles to its east?

21. At the start of stage 0, the Meta-Meme-Machine has a pool of $i = 7$ images and a pool of $t = 31$ textboxes. In each stage, it creates $i \times t$ memes by making all pairs of an image plus a textbox. The pool of images at the start of the next round consists of all previous i images as well as the $i \times t$ memes. There are still t textboxes at the start of the next round. What is the first stage s starting with a pool of more than 7 million images?

22. A positive number greater than 1 is *exponent-happy* if when written in the form $p_1^{e_1} p_2^{e_2} ... p_k^{e_k}$ for distinct primes $p_1, p_2, ..., p_k$, we have that $\gcd(e_1, e_2, ..., e_k) = 1$. How many positive numbers between 2 and 5000 inclusive are *exponent-happy*?

23. Let x, y be real numbers such that

$$x + y = 2,$$
$$x^4 + y^4 = 1234.$$

Find xy.

24. The center of a circle of radius 2 follows a path around the edges of a regular hexagon with side length 3. What is the area of the region the circle sweeps?

25. On Day 1, Aaron draws a smiley face on the board. From then on, on each day he does the same thing as the previous day (draw a smiley face or not) with probability $\frac{2}{3}$. What's the probability he draws a smiley face on Day 10?