Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Eric writes problems at a constant rate of 6 problems per hour, and Elizabeth solves problems at a constant rate of 10 problems per hour. If Eric starts writing problems 2 hours before Elizabeth starts solving problems, how many problems will he have written when Elizabeth finishes solving all the problems he has written so far?

2. Lake Donald is invaded by 1000 ducks with red or blue feathers and red or blue heads. If 538 of the ducks have red feathers, 318 ducks have blue heads and 250 ducks have both blue heads and blue feathers. How many ducks have both red heads and red feathers?

3. Maddy has a chocolate coin in the shape of a cylinder with radius 3 and height $\frac{1}{2}$. Maddy also has a gumball in the shape of a sphere with the same volume as her chocolate coin. What is the radius of her gumball?

4. Katy only owns two types of books: comic books and nature books. $1/3$ of her books are comic books. After going to a booksale, she buys 20 more comic books, so $4/7$ of her books are now comic books. How many books did she have originally?

5. A line passes through $(-2, 1)$ and $(4, 4)$. Point $(7, y)$ is also on this line. Compute y.

6. Eric has a 9-sided dice and Harrison has an 11-sided dice. They each roll their respective die. Eric wins if he rolls a number greater or equal to Harrison's number. What is the probability Eric wins?

7. A number is formed using the digits $\{2, 0, 1, 8\}$, using all 4 digits exactly once. Note that $0218 = 218$ is a valid number that can be formed. What is the probability that the resulting number is strictly greater than 2018?

8. A regular hexagon with side length 1 has a circle inscribed in it. Then another regular hexagon is inscribed in the circle. What is the area of the inner hexagon?

9. Ed writes the first 2018 positive integers down in order: $1, 2, 3, \ldots, 2018$. Then for each power of 2 that appears, he crosses out that number as well as the number 1 greater than that power of 2. After he is done, how many numbers are not crossed out?

10. The sum of the digits of Anna's age is Carly's age. In 10 years, Anna will be twice as old as Carly. What is the sum of their ages?

11. The sequence $2, 3, 5, 6, 7, 8, 10, \ldots$ contains all positive integers that are not perfect squares. Find the 2018th term of the sequence.

12. Let $ABCDE$ be a regular pentagon. Label points F on BC and G on ED such that AFG is an equilateral triangle and $FG \parallel CD$. Compute $\angle AFB$.

13. Consider the sequence $a_1 = 2$, $a_2 = 3$, $a_3 = 6$, $a_4 = 18$, $\ldots$, where $a_n = a_{n-1} \cdot a_{n-2}$. What is the largest k such that 3^k divides a_{11}?

14. Harrison the astronaut is trying to navigate his way through a rectangular grid in outer space. He starts at $(0, 0)$ and needs to reach the Earth at position $(6, 6)$. Harrison can only move upwards or rightwards. Unfortunately, there are two black holes, which are unit squares, with lower left corners at $(1, 4)$ and $(3, 1)$. If Harrison steps onto any corner of a black hole, he gets sucked in and won't be able to return home. How many paths can Harrison take to get back to Earth safely?

15. How many ways are there to select distinct integers x, y, where $1 \leq x \leq 25$ and $1 \leq y \leq 25$, such that $x + y$ is divisible by 5?

16. A turtle and rabbit both stand at the corner of a cube with side length 18 meters and race to get to the opposite corner of the cube. The turtle can swim through the interior of the cube at a rate of 6 meters/minute. The rabbit can run at a rate of 9 meters/minute, but must go along the faces of the cube. The rabbit arrives at the opposite corner first. How long must he wait for the turtle to arrive?

17. Two squares of side length 3 overlap so that the shared region is a square of side length 1. Compute the area of the smallest hexagon that covers the 2 squares.

18. How many integer pairs (a, b) satisfy $\frac{1}{a} + \frac{1}{b} = \frac{1}{2018}$?

19. A 3×3 magic square is a grid of **distinct** numbers whose rows, columns, and diagonals all add to the same integer sum. Connie creates a magic square whose sum is N, but her keyboard is broken so that when she types a number, one of the digits $(0 - 9)$ always appears as a different digit (e.g. if the digit 8 always appears as 5, the number 18 will appear as 15). The altered square is shown below. Find N.

9	11	10
18	17	6
14	11	15

20. Let $ABCD$ be a convex quadrilateral with $AB = \sqrt{2}$, $CD = 2$, and $BD = 1 + \sqrt{3}$. If $\angle ABD = 45°$ and $\angle BDC = 30°$, what is the length of AC?

21. Positive integer n has the property such that $n - 64$ is a positive perfect cube. Suppose that n is divisible by 37. What is the smallest possible value of n?

22. One of the six digits in the expression $435 \cdot 605$ can be changed so that the product is a perfect square N^2. Compute N.

23. James throws a dart at a dartboard, which is shaped like a regular hexagon of side length 2 feet. What is the probability that James throws the dart within 1 foot of any of the 6 corners of the hexagon?

24. Stu is on a train en route to SMT. He is bored, so he starts doodling in his notebook. Stu realizes that that he can combine SMT as an alphametic, where each letter represents a unique integer and the leading digits may not be zero, to get his name as shown: $\sqrt{SMT} + SMT = STU$. Find the three digit number STU.

25. Let $f(x) = x^3 - n_1x^2 + (n_2 - k^2)x - (n_3 - k^4)$. Suppose that n_1, n_2, and n_3 form a geometric sequence with common ratio k and that the roots of f are nonzero and form an arithmetic sequence with common difference also k. Find k.