Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Robin goes birdwatching one day. He sees three types of birds: penguins, pigeons, and robins. $\frac{2}{3}$ of the birds he sees are robins. $\frac{1}{8}$ of the birds he sees are penguins. He sees exactly 5 pigeons. How many robins does Robin see?

2. Jimmy runs a successful pizza shop. In the middle of a busy day, he realizes that he is running low on ingredients. Each pizza must have 1 lb of dough, $\frac{1}{4}$ lb of cheese, $\frac{1}{6}$ lb of sauce, and $\frac{1}{3}$ lb of toppings, which include pepperonis, mushrooms, olives, and sausages. Given that Jimmy currently has 200 lbs of dough, 20 lbs of cheese, 20 lbs of sauce, 15 lbs of pepperonis, 5 lbs of mushrooms, 5 lbs of olives, and 10 lbs of sausages, what is the maximum number of pizzas that Jimmy can make?

3. Queen Jack likes a 5-card hand if and only if the hand contains only queens and jacks. Considering all possible 5-card hands that can come from a standard 52-card deck, how many hands does Queen Jack like?

4. What is the smallest number over 9000 that is divisible by the first four primes?

5. A rhombus has area 36 and the longer diagonal is twice as long as the shorter diagonal. What is the perimeter of the rhombus?

6. Nick is a runner, and his goal is to complete four laps around a circuit at an average speed of 10 mph. If he completes the first three laps at a constant speed of only 9 mph, what speed does he need to maintain in miles per hour on the fourth lap to achieve his goal?

7. A fly and an ant are on one corner of a unit cube. They wish to head to the opposite corner of the cube. The fly can fly through the interior of the cube, while the ant has to walk across the faces of the cube. How much shorter is the fly's path if both insects take the shortest path possible?

8. According to Moor's Law, the number of shoes in Moor's room doubles every year. In 2013, Moor's room starts out having exactly one pair of shoes. If shoes always come in unique, matching pairs, what is the earliest year when Moor has the ability to wear at least 500 mismatched pairs of shoes? Note that left and right shoes are distinct, and Moor must always wear one of each.

9. A tree has 10 pounds of apples at dawn. Every afternoon, a bird comes and eats x pounds of apples. Overnight, the amount of food on the tree increases by 10%. What is the maximum value of x such that the bird can sustain itself indefinitely on the tree without the tree running out of food?

10. Consider a sequence given by $a_n = a_{n-1} + 3a_{n-2} + a_{n-3}$, where $a_0 = a_1 = a_2 = 1$. What is the remainder of a_{2013} divided by 7?

11. Sara has an ice cream cone with every meal. The cone has a height of $2\sqrt{2}$ inches and the base of the cone has a diameter of 2 inches. Ice cream protrudes from the top of the cone in a perfect hemisphere. Find the surface area of the ice cream cone, ice cream included, in square inches.

12. What is the greatest possible value of c such that $x^2 + 5x + c = 0$ has at least one real solution?

13. $\mathbb{R}^2$-tic-tac-toe is a game where two players take turns putting red and blue points anywhere on the xy plane. The red player moves first. The first player to get 3 of their points in a line without any of their opponent's points in between wins. What is the least number of moves in which Red can guarantee a win? (We count each time that Red places a point as a move, including when Red places its winning point.)

14. Peter is chasing after Rob. Rob is running on the line $y = 2x + 5$ at a speed of 2 units a second, starting from the point $(0, 5)$. Peter starts running t seconds after Rob, running at 3 units a second. Peter also starts at $(0, 5)$, and catches up to Rob at the point $(17, 39)$. What is the value of t?

15. Given regular hexagon $ABCDEF$, compute the probability that a randomly chosen point inside the hexagon is inside triangle PQR, where P is the midpoint of AB, Q is the midpoint of CD, and R is the midpoint of EF.

16. Eight people are posing together in a straight line for a photo. Alice and Bob must stand next to each other, and Claire and Derek must stand next to each other. How many different ways can the eight people pose for their photo?

17. An isosceles right triangle is inscribed in a circle of radius 5, thereby separating the circle into four regions. Compute the sum of the areas of the two smallest regions.

18. Caroline wants to plant 10 trees in her orchard. Planting n apple trees requires n^2 square meters, planting n apricot trees requires $5n$ square meters, and planting n plum trees requires n^3 square meters. If she is committed to growing only apple, apricot, and plum trees, what is the least amount of space, in square meters, that her garden will take up?

19. A triangle with side lengths 2 and 3 has an area of 3. Compute the third side length of the triangle.

20. Ben is throwing darts at a circular target with diameter 10. Ben never misses the target when he throws a dart, but he is equally likely to hit any point on the target. Ben gets $\lceil 5 - x \rceil$ points for having the dart land x units away from the center of the target. What is the expected number of points that Ben can earn from throwing a single dart? (Note that $\lceil y \rceil$ denotes the smallest integer greater than or equal to y.)

21. How many positive three-digit integers $\underline{a}\,\underline{b}\,\underline{c}$ can represent a valid date in 2013, where either a corresponds to a month and $\underline{b}\,\underline{c}$ corresponds to the day in that month, or $\underline{a}\,\underline{b}$ corresponds to a month and c corresponds to the day? For example, 202 is a valid representation for February 2nd, and 121 could represent either January 21st or December 1st.

22. The set $A = \{1, 2, 3, \ldots, 10\}$ contains the numbers 1 through 10. A subset of A of size n is competent if it contains n as an element. A subset of A is minimally competent if it itself is competent, but none of its proper subsets are. Find the total number of minimally competent subsets of A.

23. Let a and b be the solutions to $x^2 - 7x + 17 = 0$. Compute $a^4 + b^4$.

24. Compute the square of the distance between the incenter (center of the inscribed circle) and circumcenter (center of the circumscribed circle) of a 30-60-90 right triangle with hypotenuse of length 2.

25. A 3×6 grid is filled with the numbers in the list $\{1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9\}$ according to the following rules: (1) Both the first three columns and the last three columns contain the integers 1 through 9. (2) No number appears more than once in a given row. Let N be the number of ways to fill the grid and let k be the largest positive integer such that 2^k divides N. What is k?