Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Compute $\lim_{x\to 3} \dfrac{x^2+2x-15}{x^2-4x+3}$.

2. Compute all real values of b such that, for $f(x) = x^2 + bx - 17$, $f(4) = f'(4)$.

3. Suppose a and b are real numbers such that
$$\lim_{x\to 0} \frac{\sin^2 x}{e^{ax} - bx - 1} = \frac{1}{2}.$$
Determine all possible ordered pairs (a, b).

4. Evaluate $\displaystyle\int_0^4 e^{\sqrt{x}}\, dx$.

5. Evaluate $\lim_{x\to 0} \dfrac{\sin^2(5x)\tan^3(4x)}{(\log(2x+1))^5}$.

6. Compute $\displaystyle\sum_{k=0}^{\infty} \int_0^{\frac{\pi}{3}} \sin^{2k} x\, dx$.

7. The function $f(x)$ has the property that, for some real positive constant C, the expression
$$\frac{f^{(n)}(x)}{n+x+C}$$
is independent of n for all nonnegative integers n, provided that $n + x + C \neq 0$. Given that $f'(0) = 1$ and $\displaystyle\int_0^1 f(x)\, dx = C + (e-2)$, determine the value of C.
Note: $f^{(n)}(x)$ is the n-th derivative of $f(x)$, and $f^{(0)}(x)$ is defined to be $f(x)$.

8. The function $f(x)$ is defined for all $x \geq 0$ and is always nonnegative. It has the additional property that if any line is drawn from the origin with any positive slope m, it intersects the graph $y = f(x)$ at precisely one point, which is $\frac{1}{\sqrt{m}}$ units from the origin. Let a be the unique real number for which f takes on its maximum value at $x = a$ (you may assume that such an a exists). Find $\displaystyle\int_0^a f(x)\, dx$.

9. Evaluate $\displaystyle\int_0^{\pi/2} \frac{dx}{\left(\sqrt{\sin x} + \sqrt{\cos x}\right)^4}$.

10. Evaluate $\displaystyle\lim_{n\to\infty} \left[\left(\prod_{k=1}^{n} \frac{2k}{2k-1} \right) \int_{-1}^{\infty} \frac{(\cos x)^{2n}}{2^x}\, dx \right]$.