Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. In preparation for the annual USA Cow Olympics, Bessie is undergoing a new training regime. However, she has procrastinated on training for too long, and now she only has exactly three weeks to train. Bessie has decided to train for 45 hours. She spends a third of the time training during the second week as she did during the first week, and she spends a half of the time training during the third week as during the second week. How much time did she spend training during the second week?

2. Nick and Moor participate in a typing challenge. When given the same document to type, Nick finishes typing it 5 minutes before Moor is done. They compete again using a second document that is the same length as the first, but now Nick has to type an extra 1200-word document in addition to the original. This time, they finish at the same time. How fast (in words per minute) does Nick type? (Assume that they both type at constant rates.)

3. The Tribonacci numbers T_n are defined as follows: $T_0 = 0$, $T_1 = 1$, and $T_2 = 1$. For all $n \geq 3$, we have $T_n = T_{n-1} + T_{n-2} + T_{n-3}$. Compute the smallest Tribonacci number greater than 100 which is prime.

4. Steve works 40 hours a week at his new job. He usually gets paid 8 dollars an hour, but if he works for more than 8 hours on a given day, he earns 12 dollars an hour for every additional hour over 8 hours. If x is the maximum number of dollars that Steve can earn in one week by working exactly 40 hours, and y is the minimum number of dollars that Steve can earn in one week by working exactly 40 hours, what is $x - y$?

5. There are 100 people in a room. 60 of them claim to be good at math, but only 50 are actually good at math. If 30 of them correctly deny that they are good at math, how many people are good at math but refuse to admit it?

6. A standard 12-hour clock has hour, minute, and second hands. How many times do two hands cross between 1:00 and 2:00 (not including 1:00 and 2:00 themselves)?

7. Define a set of positive integers to be *balanced* if the set is not empty and the number of even integers in the set is equal to the number of odd integers in the set. How many strict subsets of the set of the first 10 positive integers are balanced?

8. At the 2012 Silly Math Tournament, hamburgers and hot dogs are served. Each hamburger costs \$4 and each hot dog costs \$3. Each team has between 6 and 10 members, inclusive, and each member buys exactly one food item. How many different values are possible for a team's total food cost?

9. How many ordered sequences of 1's and 3's sum to 16? (Examples of such sequences are $\{1, 3, 3, 3, 3, 3\}$ and $\{1, 3, 1, 3, 1, 3, 1, 3\}$.)

10. How many positive numbers up to and including 2012 have no repeating digits?

11. Nikolai and Wolfgang are math professors at a European university, so they enjoy researching math problems. Interestingly, each is able to do math problems at a constant rate. One day, the university gives the math department a problem set to do. Working alone, Nikolai can solve all the problems in 6 hours, while Wolfgang can solve them in 8 hours. When they work together, they are more efficient because they are able to discuss the problems, so their combined output is the sum of their individual outputs plus 2 additional problems per hour. Working together, they complete the problem set in 3 hours. How many problems are on the problem set?

12. ABC is an equilateral triangle with side length 1. Point D lies on $\overline{AB}$, point E lies on $\overline{AC}$, and points G and F lie on $\overline{BC}$, such that $DEFG$ is a square. What is the area of $DEFG$?

13. Define a number to be *boring* if all the digits of the number are the same. How many positive integers less than 10000 are both prime and boring?

14. Given a number n in base 10, let $g(n)$ be the base-3 representation of n. Let $f(n)$ be equal to the base-10 number obtained by interpreting $g(n)$ in base 10. Compute the smallest positive integer $k \geq 3$ that divides $f(k)$.

15. $ABCD$ is a parallelogram. $AB = BC = 12$, and $\angle ABC = 120°$. Calculate the area of parallelogram $ABCD$.

16. Given a 1962-digit number that is divisible by 9, let x be the sum of its digits. Let the sum of the digits of x be y. Let the sum of the digits of y be z. Compute the maximum possible value of z.

17. A circle with radius 1 has diameter AB. C lies on this circle such that $\overset{\frown}{AC} \,/\, \overset{\frown}{BC} = 4$. $\overline{AC}$ divides the circle into two parts, and we will label the smaller part Region I. Similarly, $\overline{BC}$ also divides the circle into two parts, and we will denote the smaller one as Region II. Find the positive difference between the areas of Regions I and II.

18. John is on the upper-left corner of a 3×3 grid. Once per minute, John randomly chooses a square that is either horizontally or vertically adjacent to his current square and moves there. What is the expected number of minutes that John needs to get to the center square?

19. If f is a monic cubic polynomial with $f(0) = -64$, and all roots of f are non-negative real numbers, what is the largest possible value of $f(-1)$? (A polynomial is monic if it has a leading coefficient of 1.)

20. A permutation of the first n positive integers is quadratic if, for some positive integers a and b such that $a + b = n$, $a \neq 1$, and $b \neq 1$, the first a integers of the permutation form an increasing sequence and the last b integers of the permutation form a decreasing sequence, or if the first a integers of the permutation form a decreasing sequence and the last b integers of the permutation form an increasing sequence. How many permutations of the first 10 positive integers are quadratic?

21. In trapezoid $ABCD$, $BC \parallel AD$, $AB = 13$, $BC = 15$, $CD = 14$, and $DA = 30$. Find the area of $ABCD$.

22. Two different squares are randomly chosen from an 8×8 chessboard. What is the probability that two queens placed on the two squares can attack each other? Recall that queens in chess can attack any square in a straight line vertically, horizontally, or diagonally from their current position.

23. Circle O has radius 18. From diameter AB, there exists a point C such that BC is tangent to O and AC intersects O at a point D, with $AD = 24$. What is the length of BC?

24. The quartic (4th-degree) polynomial $P(x)$ satisfies $P(1) = 0$ and attains its maximum value of 3 at both $x = 2$ and $x = 3$. Compute $P(5)$.

25. Compute the ordered pair of real numbers (a, b) such that $a < k < b$ if and only if $x^3 + \dfrac{1}{x^3} = k$ does not have a real solution in x.