Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Triangle ABC has side lengths $BC = 3$, $AC = 4$, $AB = 5$. Let P be a point inside or on triangle ABC and let the lengths of the perpendiculars from P to BC, AC, AB be D_a, D_b, D_c respectively. Compute the minimum of $D_a + D_b + D_c$.

2. Pentagon $ABCDE$ is inscribed in a circle of radius 1. If $\angle DEA \cong \angle EAB \cong \angle ABC$, $m\angle CAD = 60°$, and $BC = 2DE$, compute the area of $ABCDE$.

3. Let circle O have radius 5 with diameter $\overline{AE}$. Point F is outside circle O such that lines $\overline{FA}$ and $\overline{FE}$ intersect circle O at points B and D, respectively. If $FA = 10$ and $m\angle FAE = 30°$, then the perimeter of quadrilateral $ABDE$ can be expressed as $a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6}$, where a, b, c, and d are rational. Find $a + b + c + d$.

4. Let ABC be any triangle, and D, E, F be points on $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ such that $CD = 2BD$, $AE = 2CE$ and $BF = 2AF$. $\overline{AD}$ and $\overline{BE}$ intersect at X, $\overline{BE}$ and $\overline{CF}$ intersect at Y, and $\overline{CF}$ and $\overline{AD}$ intersect at Z. Find $\frac{Area(\triangle ABC)}{Area(\triangle XYZ)}$.

5. Let $ABCD$ be a cyclic quadrilateral with $AB = 6$, $BC = 12$, $CD = 3$, and $DA = 6$. Let E, F be the intersection of lines AB and CD, lines AD and BC respectively. Find EF.

6. Two parallel lines l_1 and l_2 lie on a plane, distance d apart. On l_1 there are an infinite number of points $A_1, A_2, A_3, \cdots$, in that order, with $A_nA_{n+1} = 2$ for all n. On l_2 there are an infinite number of points $B_1, B_2, B_3, \cdots$, in that order and in the same direction, satisfying $B_nB_{n+1} = 1$ for all n. Given that A_1B_1 is perpendicular to both l_1 and l_2, express the sum $\sum_{i=1}^{\infty} \angle A_iB_iA_{i+1}$ in terms of d.

7. In a unit square $ABCD$, find the minimum of $\sqrt{2}AP + BP + CP$ where P is a point inside $ABCD$.

8. We have a unit cube $ABCDEFGH$ where $ABCD$ is the top side and $EFGH$ is the bottom side with E below A, F below B, and so on. Equilateral triangle BDG cuts out a circle from the cube's inscribed sphere. Find the area of the circle.

9. We have a circle O with radius 10 and four smaller circles O_1, O_2, O_3, O_4 of radius 1 which are internally tangent to O, with their tangent points to O in counterclockwise order. The small circles do not intersect each other. Among the two common external tangents of O_1 and O_2, let l_{12} be the one which separates O_1 and O_2 from the other two circles, and let the intersections of l_{12} and O be A_1 and B_2, with A_1 denoting the point closer to O_1. Define l_{23}, l_{34}, l_{41} and $A_2, A_3, A_4, B_3, B_4, B_1$ similarly. Suppose that the arcs A_1B_1, A_2B_2, and A_3B_3 have length π, $3\pi/2$, and $5\pi/2$ respectively. Find the arc length of A_4B_4.

10. Given a triangle ABC with $BC = 5$, $AC = 7$, and $AB = 8$, find the side length of the largest equilateral triangle PQR such that A, B, C lie on QR, RP, PQ respectively.